

Cluster temperature profiles and Sunyaev-Zeldovich observations

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ABSTRACT

Galaxy clusters are not iso-thermal, and the radial temperature dependence will affect the cluster parameters derived through the observation of the Sunyaev-Zeldovich (SZ) effect. We show that the derived peculiar velocity will be systematically shifted by 10 – 20%. For future all-sky surveys one cannot rely on the observationally expensive X-ray observations to remove this systematic error, but one should instead reach for sufficient angular resolution to perform a deprojection in the SZ spectra. The Compton weighted electron temperature is accurately derived through SZ observations.

Key words: galaxies: clusters: general — galaxies: structure

1 INTRODUCTION

Galaxy clusters have been known and studied for many years, and the radial dependence of cluster temperatures is becoming a testing ground for models of structure formation and for our understanding of gas dynamics. Galaxy clusters are not iso-thermal. Instead, the emerging temperature profile is one where the temperature is approximately flat or increases from the centre to some characteristic radius, and then decreases again for larger radii.

The central temperature decrement has been much discussed and the possibility of cooling flows has been explained in excellent reviews (Fabian 1994; Donahue & Voit 2003). Many clusters are well fit with a power law $T \sim r^\tau$ in the very central region (Voigt & Fabian 2003), with a slope, τ , between 0.15 and 0.45. Numerical simulations are only now beginning to see this central decrement (see Motl et al. (2003) for references).

The outer temperature decrease is well established both observationally (Markevitch et al. 1998; De Grandi & Molendi 2002; Kaastra et al. 2003; Arnaud, Pratt & Pointecouteau 2003) and numerically (see Lin et al. (2003) and references therein). The simulations have even started to provide estimates of the outer temperature decrement which are well fit with power laws in fair agreement with observations.

Such non-trivial temperature profiles also affect derived cosmological parameters like the Hubble parameter (Battistelli et al. 2003; Lin, Norman & Bryan 2003).

On the other hand, observations of the Sunyaev-Zeldovich (SZ) effect are becoming increasingly accurate (Laroque et al. 2002; De Petris et al. 2002; Battistelli et al. 2003), however, most analyses of SZ observational data are made under the simplifying assumption of iso-thermality.

Several groups have considered the ability of future SZ observations to extract cluster parameters (Knox, Holder & Church 2003; Aghanim, Hansen & Lagache 2004), but always under the assumption of iso-thermality. We therefore set out to study the effect on the SZ derived parameters of non-trivial cluster temperature profiles.

In section 2 we show that the relevant quantities for SZ observations are Compton-averaged, in particular we emphasize the difference between the temperatures derived through X-ray and SZ observation for clusters which are not iso-thermal. In section 3 we use observed and simulated cluster profiles to show that the systematic shift in the derived peculiar velocity, v_p , is of the order 10 – 20%. Finally we comment on the possibility of deprojection of SZ observed spectra in section 4.

2 THE SZ EFFECT

The SZ effect is dominated by the Compton parameter

$$y = \sigma_T \int n_e \frac{kT_e}{m_e c^2} dl, \quad (1)$$

where σ_T is the Thomson cross section, n_e , T_e and m_e are number density, temperature and mass of the electrons, and the integral is along the line of sight. If we extract the radial dependence of the parameters (Birkinshaw, Hughes & Arnaud 1991)

$$\begin{aligned} n_e(r) &= n_e^0 f_{n_e}(r), \\ T_e(r) &= T_e^0 f_{T_e}(r), \end{aligned}$$

then y can be written as $y = \kappa_y \int f_{n_e} f_{T_e} dl$, where the constant is $\kappa_y = \sigma_T n_e^0 k T_e^0 / (m_e c^2)$. It will be convenient to introduce $f_y = f_{n_e} f_{T_e}$, in which case we have

$$y = \kappa_y \int f_y dl. \quad (2)$$

Similarly, the optical depth is

$$\tau = \kappa_\tau \int f_{n_e} dl, \quad (3)$$

with $\kappa_\tau = \sigma_T n_e^0$, and one sees that for an iso-thermal cluster, where $f_{T_e}(r) \equiv 1$, one has $y \sim \tau T_e^0$.

For any cluster observation one receives information from an integral along the line of sight. Since the radial structures of clusters are non-trivial this implies that one may have to define averaged quantities. For X-ray observations the averaging procedure is often simplified to emission-weighted quantities, scaled by the emissivity which for large temperatures roughly scales (Sarazin 1986) as $\epsilon \sim n^2 T_e^{1/2}$

$$\langle \mathcal{O} \rangle_X = \frac{\int \mathcal{O} \epsilon dl}{\int \epsilon dl}. \quad (4)$$

For the SZ effect the relevant scale is not the emissivity, but instead the local Compton parameter, $n_e(r)T_e(r)$, and therefore the average of \mathcal{O} is given by

$$\langle \mathcal{O} \rangle_y = \frac{\int \mathcal{O} f_y dl}{\int f_y dl}, \quad (5)$$

for instance, the average SZ temperature for a cluster is

$$\langle T_e \rangle_y = \frac{\int T_e f_y dl}{\int f_y dl} = \frac{\kappa_y \int T_e f_y dl}{y}. \quad (6)$$

We will hereafter only discuss Compton-averaged quantities, and hence omit the index on averages.

The SZ effect for an iso-thermal cluster is composed of several parts

$$\frac{\Delta I}{I_0} = y (g(x) + T_e \delta(x, T_e)) - \beta_p \tau a(x), \quad (7)$$

where $\beta_p = v_p/c$ is the bulk motion of the cluster referred to as the peculiar velocity. The spectral forms due to the thermal up-scattering, $g(x)$, and due to the overall cluster motion, $a(x)$, are well-known (Sunyaev & Zel'dovich 1972; Sunyaev & Zel'dovich 1980), and the spectral form of the relativistic corrections to the thermal up-scattering, $\delta(x, T_e)$, is easily calculated (Wright 1979; Rephaeli 1995; Dolgov et al. 2001; Itoh & Nozawa 2003; Shimon & Rephaeli 2004). To a very good approximation $\delta(x, T_e)$ can be taken as independent of temperature (see e.g. Diego et al. (2002)), that is, for a given frequency, x , one can write $\delta(x, T_e) = b_1(x) + b_2(x)T_e + \mathcal{O}(T_e^2)$, where b_2 and higher order terms are subdominant. For the present discussion we can ignore a contribution to eq. (7) of the marginally detectable ultra-relativistic electrons (Ensslin & Kaiser 2000; Ensslin & Hansen 2004), since our findings will not depend on them. For excellent reviews on the SZ effect see (Birkinshaw 1999; Carlstrom, Holder & Reese 2002).

For a cluster with a non-trivial temperature profile eq. (7) is written as an integral along the line of sight

$$\frac{\Delta I}{I_0} = \int [\kappa_y f_y (g(x) + T_e^0 f_{T_e} \delta(x)) - \beta_p \kappa_\tau f_{n_e} a(x)] dl, \quad (8)$$

where the quantities $f_{n_e}(r)$ and $f_{T_e}(r)$ are the local quantities. Now, using the definition in eq. (5), this means

$$\frac{\Delta I}{I_0} = y [g(x) + \langle T_e \rangle \delta(x) - \beta_p \langle 1/T_e \rangle (m_e c^2/k) a(x)] \quad (9)$$

For an iso-thermal cluster, where $f_{T_e} \equiv 1$, one has $1/\langle T_e \rangle = \langle 1/T_e \rangle$, but for a thermally non-trivial cluster there are really 4 independent variables, $y, \beta_p, \langle T_e \rangle$ and $\langle 1/T_e \rangle$. Clearly, for such a cluster the X-ray derived temperature, T_X , may differ significantly from $\langle T_e \rangle$ (as well as, $1/T_X \neq \langle 1/T_e \rangle$), and hence the X-ray derived temperature should only be used with great caution in the study of SZ observations.

Most studies of SZ observations include up to 2 free parameters (Laroque et al. 2002; De Petris et al. 2002; Battistelli et al. 2003), for instance the Compton parameter and the peculiar velocity, and only very few have attempted studies with 3 parameters (Hansen, Pastor & Semikoz 2002) with inclusion of the temperature. We will here consider the effect of neglecting the non-trivial radial temperature profile, and show that it induces a systematic error on the peculiar velocity of the order 10%.

3 CLUSTER PROFILES

As a specific example we use a simple cluster model, where the electron density profile follows a β -model

$$n_e(r) = n_e^0 \left(1 + \left(\frac{r}{r_c} \right)^2 \right)^{-\beta_{3/2}}, \quad (10)$$

where r_c is a characteristic radius. Such broken power-law density profiles are often used to fit observations (Cavaliere & Fusco-Femiano 1976; Lauer et al. 1995; Lewis, Buote & Stocke 2003), and are possibly even understood theoretically (Hansen & Stadel 2003; Hansen 2003; Ruskowski, Bruggen & Begelman 2004), but may need slight modifications for non-isothermal clusters (Ettori 2000). The temperature profile may have the shape

$$T_e(r) = T_e^0 \left(1 + \frac{r}{r_c} \right)^{-\alpha}, \quad (11)$$

where we use results from recent numerically simulated clusters (Lin, Norman & Bryan 2003) with $\alpha = 0.56$ and $\beta = 0.61$. For cooling flow clusters the form is slightly more complicated, but an inclusion of the central decrement is straight forward. With these profiles one finds the average temperature from eq. (6) to be $\langle T_e \rangle = 7.08$ keV when the central temperature is 10 keV, and $\langle 1/T_e \rangle \langle T_e \rangle = 1.12$ which indicates that one will find a peculiar velocity which is approximately 12% systematically too large when using SZ observations to define the cluster parameters. We will now test these findings numerically, by constructing and analysing the true SZ signal from this cluster model.

With this cluster model, where we normalize the optical depth to $\tau = 0.01$, we construct the SZ signal along the line of sight through the cluster centre. As a specific example we take an observation with 4 observing frequencies at 90, 150, 220 and 270 GHz, and assume a sensitivity of $1 \mu K$ for each channel. We choose $v_p = 1000$ km/sec. For optimal observing frequencies (Aghanim et al. 2003; Holder 2002) the statistical error-bars can be reduced slightly (represented by $\Delta\chi^2 = 1$ on figure 1), but the systematic shift (about 12%) will remain the same. The systematic shift is independent of the assumed sensitivity, which here is taken slightly optimistic (ACT expects a sensitivity of $2 \mu K$).

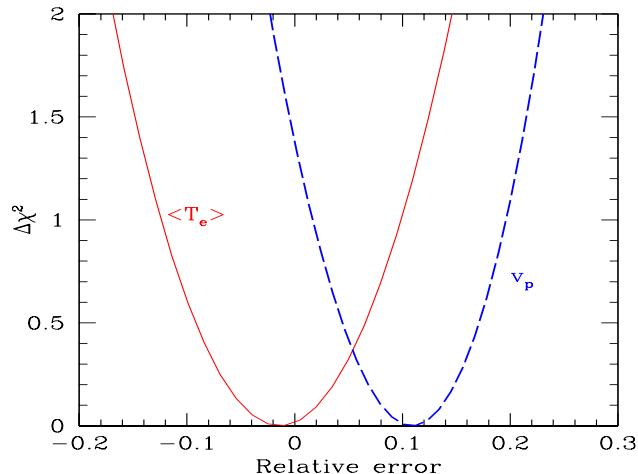


Figure 1. The relative error on temperature, $\langle T_e \rangle$, and peculiar velocity, v_p , through observation of a model SZ cluster with non-trivial temperature profile (eq. (11) with $\alpha = 0.56$) and a β -profile for the electron density (eq. 10 with $\beta = 0.61$). The SZ-derived temperature (solid, red line) is very close to the Compton weighted temperature, but the peculiar velocity (dashed, blue line) is systematically shifted by 10 – 12%.

One can now treat this constructed SZ signal (and corresponding error-bars) as a real observation, and derive the 3 cluster parameters, y, T_e, v_p in the standard way. This is simply done by inputting the constructed SZ signal into the publicly available ^{*} SZ parameter extraction code `sasz` (Hansen 2004). The result is shown in figure 1, where we plot both peculiar velocity and temperature on the same figure. The solid (red) line is the relative error on the temperature, $(T_e - 7.08)/7.08$, and the dashed (blue) line is the relative error on the peculiar velocity, $(v_p - 1000)/1000$. As is clear, the average temperature is indeed very near the expected 7 keV, whereas the peculiar velocity is shifted by 10 – 12%.

The fact that the SZ determined temperature indeed turns out to be the “correct” Compton weighted temperature implies that accurate prior temperature knowledge (e.g. from X-ray observations) would not change the systematic shift of the peculiar velocity. Having both accurate temperature and electron density profiles from X-ray observations would allow one to calculate the systematic shift (ignoring the complications from an unknown clumping). However, peculiar velocities are most important when derived for a large sample of clusters (e.g. from an all-sky SZ survey), but detailed X-ray maps are observationally expensive and cannot be made for the large number of clusters expected in future all-sky SZ surveys.

The minimal deviation from the derived central value of the temperature arises because $\delta(x)$ in eq. (8) is not completely independent of the temperature, but this minor effect

is not important for our discussion. Furthermore, if one imposes a “known” temperature of $T_e = T_X = 10$ keV (from X-ray observations) then that would lead to a $\sim 40\%$ systematic shift of v_p , and even a “known” emission weighted temperature of $T_X \approx 8$ keV would give a $\sim 20\%$ systematic error on v_p . To be very explicit, this means that if one knows the exact temperature and electron density profiles from X-ray observations, and then uses the emission weighted temperature in the analysis of SZ observations, then one would overestimate the peculiar velocity with 20% for this particular cluster. If a given observation only puts bounds on the peculiar velocity, then the corresponding error-bars are overestimated by the same amount. This shows, as mentioned earlier, that X-ray observed temperatures should be used with great caution in the study of SZ observations. If one does know the exact temperature and density profiles, then clearly one can calculate $\langle 1/T_e \rangle$, with which one can extract the peculiar velocity correctly.

We thus see that for this specific choice of temperature and electron density profiles the systematic shift in the derived peculiar velocity from SZ observations is 10 – 12%. The question now arises: which range in systematic shift is expected for real clusters?

The density profiles of many clusters are well described by a β -model with $\beta \sim 0.67$, however, numerical simulations show a rather large scatter in the numerical value for β . Fixing the temperature profile to the one in eq. (11) with $\alpha = 0.56$, and letting β vary from $\beta = 0.75$ to $\beta = 0.46$ (Lin, Norman & Bryan 2003) leads to systematic shifts in the peculiar velocity of 8% to 20%. One could instead fix the density slope to $\beta = 2/3$, and use a polytropic equation of state

$$T_e = T_e^0 \left(\frac{n_e}{n_e^0} \right)^{\gamma-1}, \quad (12)$$

with γ in the range 1 to 5/3. Even though the polytropic shape does not provide an excellent fit to observations (De Grandi & Molendi 2002), it is sufficient for estimating the magnitude of the systematic error. Fitting observations with eq. (12) gives $\gamma = 1.46 \pm 0.06$ for non-cooling flow clusters, and $\gamma = 1.20 \pm 0.06$ for cooling flow clusters (De Grandi & Molendi 2002). With $\gamma = 1.46$ one gets a systematic shifts of the peculiar velocity of 20% (for $\gamma = 1.20$ one gets 6%), and for the theoretically extreme case of an adiabatic gas with $\gamma = 5/3$ the error becomes about 30%. A simple fit to the error in percent of the peculiar velocity as a function of γ , which is accurate for $\gamma > 1.2$, gives: error (in %) = $53(\gamma - 1) - 5$.

Cluster merging induces bulk motion within the cluster as large as 500 km/sec (see e.g. Sunyaev et al. (2003)), and the corresponding mis-estimate of the peculiar velocity is about 10% (Haehnelt & Tegmark 1980), or up to about 100 km/sec (Holder 2002).

4 ONION PEELING A SZ CLUSTER

We have seen above that the SZ determined peculiar velocity of realistic galaxy clusters may be systematically wrong by 10 – 20%, and that X-ray observations cannot be expected to solve this problem because they are observationally expensive. The natural question is then: what is the solution?

^{*} <http://krone.physik.unizh.ch/~hansen/sz/>

This SZ parameter extraction code uses the Monte Carlo like method of simulated annealing to derive the central SZ parameters and corresponding error-bars.

For X-ray observations the similar problem is solved by the technique of *deprojection*. This heuristically corresponds to peeling an onion. The outermost layer is observed and analysed. Then the next layer is analysed while subtracting the signal from the outer layer. The error-bars will increase for the inner layers, but for X-rays this turns out not to be disastrous. That is because the contribution from the outer layers is significantly smaller than the contribution from the inner layers, since the emission drops fast with radius $\epsilon \sim T^{1/2} n^2 \sim r^{\zeta_X}$ with $\zeta_X \sim -4.2$.

Similarly, for SZ observations the relevant quantity is the local Compton parameter, which goes like $n_e T_e \sim r^{\zeta_y}$, with $\zeta_y \sim -2.5$. Thus, the error-bars should remain under control even when subtracting contribution to the intensity from outer layers. Naturally, the fact that $2.5 < 4.2$ is the reason why SZ observations are more suitable than X-ray observations for measuring the outer cluster region.

An advantage for SZ deprojection is that the peculiar velocity is the same for the entire cluster when we can ignore the local gas motion (see however Holder (2002)). That means that while normally one would expect to have 3 SZ parameters (y, T_e, v_p) for each of the N radial bins, giving $3N$ parameters, then we really only have $2N + 1$ free parameters. This is important because the major parameter degeneracy for SZ observations is between the temperature and peculiar velocity. Thus, the deprojection itself may help reduce the error-bars on the derived parameters. It is worth mentioning that spatially resolved SZ observations will provide both temperature and electron density profiles, so combined with accurate X-ray observations one can directly measure the radial dependence of clumpiness, which is a non-trivial measure of the merger history.

Such SZ deprojection of the observed intensity clearly demands that the future SZ observations should both have good *spectral coverage* and good *spatial resolution*. That is a non-trivial observational challenge.

5 CONCLUSION

We have shown that due to the non-trivial temperature structure of galaxy clusters, the SZ derived peculiar velocity will be systematically shifted by 10 – 20%. The Compton weighted electron temperature is, however, derived accurately. For future all-sky SZ surveys one cannot rely on the observationally expensive X-ray observations to remove this systematic error, but one should instead reach for sufficient angular resolution to perform a deprojection in the SZ spectra.

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